Synchronization properties of two self-oscillating semiconductor lasers subject to delayed optoelectronic mutual coupling

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We theoretically investigate the nonlinear dynamics and synchronization properties between two mutually coupled semiconductor lasers units. Each unit can self-oscillate by means of delayed optoelectronic feedback loops. The mutual optoelectronic interactions between the laser units take into account the finite propagation time of the signals. Under perfectly symmetric conditions, we find different "death by delay" islands that persist for instantaneous coupling. The appearance of (zero lag) isochronous chaotic synchronization, under appropriate driving conditions, is another distinctive feature of the delayed feedback loops in the laser units. For slightly asymmetric operation, we obtain frequency locked bands (Arnold Tongue) whose width periodically changes with the coupling delay time.

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The inclusion of delay times in the interaction between different units of a physical system has been found to be responsible for a number of unexpected behaviors yielding important consequences in the theory of control and stabilization processes [1,2]. These delays, which naturally arise because of the finite propagation speed of the signals, not only introduce a phase space of infinite dimension but they also give rise to new phenomena and applications.

Semiconductor lasers represent ideal candidates for exploring these phenomena when they are coupled or subject to external perturbations, due to their inherent nonlinearity and rich dynamics. In particular, the concepts of synchronization were applied to the laser system already in 1994 [3]. So far, a variety of coupling configurations have been addressed in the literature. The interest in unidirectionally coupled semiconductor lasers has grown since the suggestion that they might be used in encoded optical communication systems [4–11]. The dynamics of mutually coupled lasers via the coherent optical field injection [12–15] or by an optoelectronic coupling [16–18] has also been a subject of intense research.

Recently, the authors have experimentally investigated [19] the effect of including an internal optoelectronic feedback loop within each mutually coupled laser unit. Different types of dynamical behaviors such as chaos synchronization and the appearance of the "death by delay" phenomenon were reported. The results presented in Ref. [19] describe interesting phenomena, although, the underlying physical mechanisms have not been explored yet. This report provides a theoretical understanding of the above-mentioned experimental findings, being complemented with the study of two slightly asymmetric laser units.

Our aim is to study the nonlinear dynamical behavior and the synchronization properties of two single-mode semiconductor lasers, including feedback loops, subject to optoelectronic mutual coupling. In this configuration, the uncoupled laser units exhibit oscillatory, pulsating, or even chaotic behavior, depending on the strength and delay time that characterize their own feedback loops. Specifically, we investigate the phenomenon of oscillation death, in which the coupling between the two oscillators induces the quenching of the limit cycle solutions through a collapse to the zero amplitude state [19–23]. This quenching induced by the delayed interaction, commonly known in the literature as "death by delay," is naturally observed in our system of mutually coupled lasers. When looking at the synchronization properties of our two lasers system we observe, under appropriate conditions, the appearance of isochronal chaotic synchronization between the laser intensities. Interestingly, this solution is unstable when the laser units do not include feedback loops [17]. Another important aspect in coupled systems is their locking behavior. In particular, the frequency locking properties of two similar coupled oscillators is a subject of wide interest both theoretically and in practical applications. Here, we analyze the effect of the coupling delay time on the locking properties.

We consider a system composed by two identical single-mode semiconductor lasers subject to optoelectronic coupling and feedback, as in the experiments described in Ref. [19]. The optical power emitted by each laser is detected, amplified, and added to the bias current of its counterpart (optoelectronic coupling) and to its own injection current (optoelectronic feedback). These terms are delayed due to the finite propagation time of the optical and electrical signals. The dynamics of the photon and carrier densities is described by the single-mode semiconductor laser rate equations appropriately modified in order to include the coupling and feedback loops. The photon and carrier densities S_j and N_j are governed by

$$\frac{dS_j}{dt} = [\Gamma g_j - \gamma_c] S_j, \tag{1}$$

$$\frac{dN_j}{dt} = \frac{I_{b,j} + \Delta I_j}{Ve} - \gamma_{sj} N_j - g_j S_j, \tag{2}$$

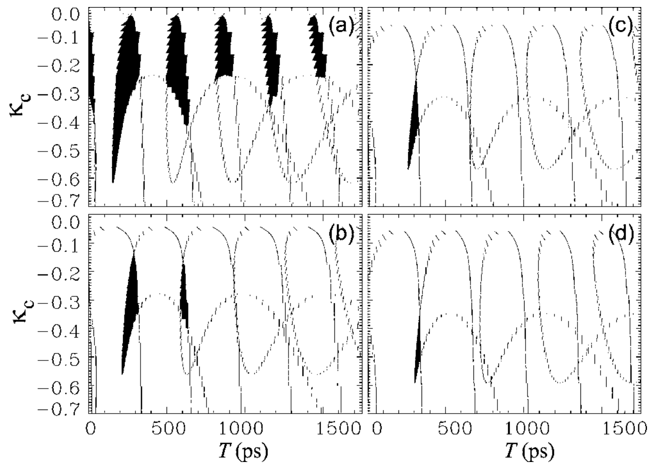

FIG. 1. Hopf curves and "death islands" (shaded regions) in the κ_c-T plane for FP4 for several values of the feedback delay time 975 ps (a), 1050 ps (b), 1125 ps (c), 1200 ps (d). The feedback strength is $\kappa_f=0.3$ and the bias current is set to 10% above threshold.

$$\Delta I_j = e\gamma_c[\kappa_{fj}S_j(t-\tau_j) + \kappa_{c3-j}S_{3-j}(t-T_{3-j})], \tag{3}$$

$$g_j \approx g_0 + g_n(N_j - N_0) + g_p(S_j - S_0), \tag{4}$$

where $j=1,2$ labels each laser. The physical meaning and numerical values of the different parameters are given in Ref. [16]. κ_c and κ_f stand for the coupling and feedback rates, respectively. T_j is the delay in the coupling lines between lasers whereas τ_j is the delay time of the feedback loops.

We start assuming two device-identical lasers operating under symmetric conditions, i.e., identical bias currents, coupling strengths, feedback strengths, and feedback loop delays in both lasers are considered. Under these conditions the system has four fixed point solutions. The first solution (FP1) defines the "off" state of the lasers. It becomes unstable when the bias current exceeds the solitary laser threshold. Two additional fixed points (FP2 and FP3) correspond to the case where one laser is switched on and the other one switched off. These solutions represent the only two possible asymmetric steady states. Finally, the steady-state FP4 is symmetric and defines the "on" state of both lasers, whose stability properties will be analyzed next.

A proper combination of the feedback strength and feedback delay times allows us to prepare each laser as a limit cycle oscillator when uncoupled. Under this situation, we inspect the role of the coupling strength κ_c and the coupling delay time $T=(T_1+T_2)/2$. We remark that T is the relevant bifurcation parameter, instead of T_1 and T_2, since only the sum of these delays appears in the characteristic equation [16]. Figure 1 shows the Hopf curves for the fixed point named FP4 in the κ_c vs T plane. By studying the direction of transition of the eigenvalues when crossing the imaginary axis at a Hopf curve, we are able to find closed regions in the parameter space $\kappa_c - T$ where the FP4 is stable, thus inducing the death of the oscillations. Shaded regions in Fig. 1, surrounded by supercritical Hopf lines, define the "death islands" for a feedback strength $\kappa_f=0.3$. Different panels in the figure show the death islands for feedback delay times ranging from $\tau=975$ to $\tau=1200$ ps.

We find that the "death by delay" phenomenon appears in a wide parameter range in our system. For this value of κ_f and feedback delay times shorter than $\tau=925$ ps, not shown in the figure, no death islands can be defined since the lasers are already stable even when decoupled. Death islands start to appear when the solitary lasers undergo self-sustained oscillations at $\tau \sim 950$ ps. Several death islands computed for $\tau=975$ ps can be observed in Fig. 1(a), which are regularly, although not completely, spaced. The existence of multiple islands when varying T has been experimentally found in Ref. [19]. It can be also noticed in Fig. 1 that the size of these islands decreases when the coupling delay time T increases, until they disappear for $T \gtrsim 1500$ ps. Interestingly, both the number of islands and their size continuously decrease when increasing τ, until they disappear for $\tau = 1225$ ps. In the regions surrounding the death islands the system operates in a limit cycle.

A surprising feature in Fig. 1(a) is the fact that one of the death islands reaches the $T=0$ axis. We have checked that this fact occurs for feedback delays τ in the range 950–1025 ps. Therefore there is an apparent contradiction with the argument that no identical oscillators can drive each other to a zero amplitude state in the absence of delay in the coupling [20,21]. The controversy arises from the special origin of the pulsating behavior in our laser system, which is induced by feedback loops with characteristic delay times. It is worth noting that the preceding studies of death by delay [20,21] considered systems containing only one time delay. Our results indicate that in order to induce the amplitude quenching effect between coupled oscillators neither an asymmetry nor a delayed coupling is strictly necessary if the oscillators are subject to delayed feedback loops. In general, we conjecture that a delay term, regardless of its origin, is necessary in the system in order to observe the "death" of the amplitude oscillations in identically coupled oscillators. As a final remark we would like to comment on the bandwidth filtering effects introduced by the optoelectronic components in the experiments. In general, the main qualitative features of the death islands shown in Fig. 1 remain even for finite bandwidths. The effects of the high cutoff frequency of the filter are not important when this cutoff frequency is higher than the oscillation frequencies (as mostly happens in the experiments). On the contrary the low cutoff frequency filters the continuous part of the spectrum what entails to an increase of the size of the death islands and to the creation of new ones.

Experimental studies [19] have demonstrated the existence of (zero-lagged) isochronal chaotic synchronization between the lasers when the feedback loops are included. To check whether isochronal synchronization appears also in our model, we operate the lasers in a chaotic regime by increasing the feedback strength and delay time, so they exhibit chaotic oscillations even when uncoupled. In Fig. 2(a) shows the time series of the normalized output power, Fig. 2(b) the synchronization plot, i.e., the output power of one laser against the power of the other one, and Fig. 2(c) the cross-correlation function between the laser intensities. It can

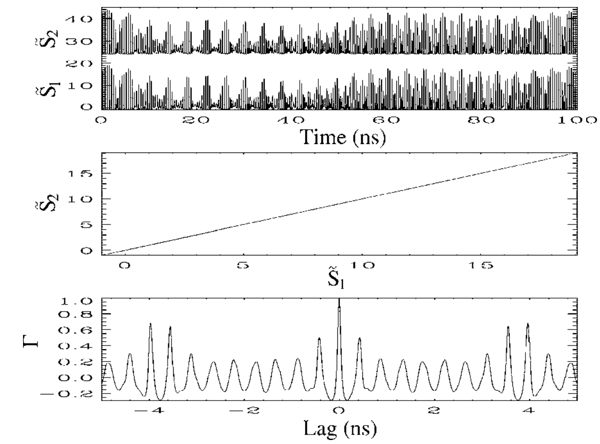

FIG. 2. Top: chaotic temporal series of the lasers intensities after coupling; middle: synchronization plot; bottom: cross-correlation function Γ between the two chaotic outputs. When uncoupled, both lasers operate in the chaotic regime due to their feedbacks loops with κ_f=0.4 and τ=3.5 ns. The coupling strength is κ_c=0.05 while the coupling delay is T=3.85 ns.

be observed that the synchronization plot displays an almost perfect 45° line while the maximum of the cross-correlation function occurs at zero time shift between the intensity signals. These results demonstrate the existence of isochronal chaotic synchronization in some parameters range, in agreement with the experimental findings. It is worth mentioning that this zero lag synchronization is lost if the feedback delay time and the coupling time are very different.

We now turn into the problem of slightly asymmetric laser units. When both lasers are operated in a pulsating regime, one of the feedback delay times is slightly changed with respect to the other one. This change, that we identify as a detuning, induces different natural pulsation frequencies. It is well known in the literature that two coupled oscillators with different frequencies can lock to each other in some regions usually called Arnold tongues. In our case, the Arnold tongues define a region in the coupling strength versus detuning space where the intensity oscillations of both lasers lock to the same frequency. In this work, we focus on the dependence of the Arnold tongues on the coupling delay time between the lasers. The numerically computed main (1:1) Arnold tongue is shown in Fig. 3(a). We found that a change in the coupling delay time induces a change in the width of this tongue. The dependence of this width when the coupling delay time is changed is shown in Fig. 3(b) for a coupling coefficient κ_c=0.08. It can be seen that the width of the Arnold tongue displays repetitive variations whose period is close to one-half of the period of the intensity oscillations. The salient feature is the capability of the coupling delay time to enhance the width of the Arnold tongue by a factor larger than 2.

To complete the study of the Arnold tongue we have analyzed the mechanisms underlying the repetitive variations in the locking width. A similar mechanism has been observed in a configuration of two delayed coupled oscillators described by the Kuramoto model. In this case, the fact that the locking width can exceed the one for the zero-delay case is associated to a frustration phase parameter [24]. In our case, the

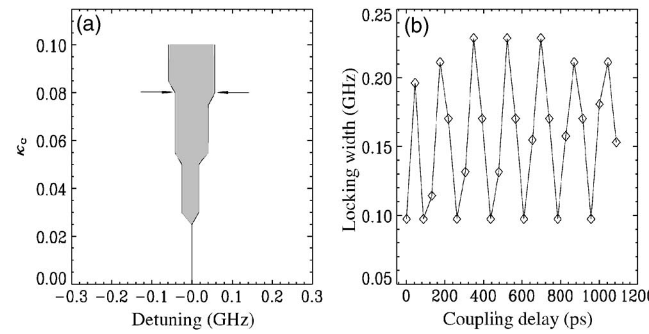

FIG. 3. (a) The main Arnold tongue for T=0. (b) Dependence of the locking width with the coupling delay time for κ_c=0.08. The feedback strength is κ_f=0.3, and the bias current is 33% above threshold. The natural period of the oscillations when the lasers are uncoupled is about 348 ps.

variation of the Arnold tongue width can be understood as follows. In the absence of coupling, the electrical feedback loops generate gain modulation in the lasers. If we couple two of these lasers unidirectionally in a master-slave configuration, the slave laser locks to the externally imposed clock, and the coupling delay time only imposes a relative phase to the oscillations. We note, however, that may exist a certain time shift between the emission of the optical pulse and the externally injected electrical signal since the process is mediated by the dynamics of the carrier reservoirs in the active region of the lasers; the electrical injection from the coupling slightly modifies the gain modulation created by the feedback loops. This process allows for small adjustments of the repetition rate through small temporal shifts of the pulses. Now, the role of the coupling delay time becomes significant in the case of bidirectional coupling since it can enforce the locking of the oscillations. A necessary condition for periodic locking is that the time required by a pulse to travel along the complete path and returning to a given reference point must be an integer number of the period of the locked oscillations. This time comprises the total coupling time $2T$ and the (small) nonlinear time shifts introduced by the lasers ΔT_1 and ΔT_2. Hence, the locking condition can be written as $2T+\Delta T_1+\Delta T_2=nT_{osc}$, with n an integer and T_{osc} the period of the oscillations. This condition has a repetitive structure when T is changed by $\sim T_{osc}/2$, which is reproduced in the oscillations of thelocking width [Fig. 3(b)]. We have seen that the allowed values of $\Delta T_1+\Delta T_2$ as a function of the detuning is limited in a certain interval given by the actual coupling strength. This limited tunability of $\Delta T_1+\Delta T_2$ leads to the conclusion that there exist some values of T for which the locking condition can be more easily satisfied (leading to large Arnold tongues). It is worth noting that this effect can be exploited in any possible application where a robust locking state between lasers is required.

In summary, we have theoretically investigated the nonlinear dynamics and synchronization properties of two bidirectionally coupled semiconductor lasers subject to optoelectronic feedback loops. We have concentrated on the synchronization properties of the lasers when they operate as either limit cycle or chaotic oscillators. The death by delay phenomenon has been characterized and compared with previously reported experimental results. A different scenario for the quenching of the oscillations of two identical units that occurs in the absence of delay time in the coupling line is found. We attribute this interesting behavior to the inclusion of delayed feedback loops, which physically act as an additional memory effect in the system. When operating in a chaotic regime, a zero lag identical synchronization solution is obtained if the feedback delay times are close to the coupling time. If this condition is not satisfied, the zero lag solution becomes unstable.

We have also considered slightly mismatched operation of the lasers; the feedback delay time of one of the lasers is changed to induce a small detuning between the oscillations. Under this condition we have determined the locking regions between the two oscillators. We have found that the size of this locking region varies regularly when changing the coupling delay time with a period close to half the period of the laser intensity oscillations. Since the locking width can be increased beyond the zero-delay case, a delay time in the coupling between two units could be exploited to enhance the locking stability for some systems.

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